Lecture 19
Optical MEMS (1)

Agenda:

- Optics Review

Optics Review

- Nature of Light
- Reflection and Refraction
- Total Internal Reflection
- Lenses
- Numerical Aperture
- Diffraction
- Polarization
- Interference
- Doppler Effect
- Coherence
- Optic Fibers: Basics
The Nature of Light

- **Particle** – Light consists of photons
- **Wave** – Light travels as a transverse electromagnetic wave
- **Ray Theory** – Light travels along a straight line and obeys laws of *geometrical optics*. Ray theory is valid when the objects are much larger than the wavelength.

Spherical and plane wave fronts

A wave front is the locus all points in the wave train which have the same phase.
Field Distribution of Plane Wave

Electric and magnetic fields are orthogonal to each other and to the direction of propagation \( z \).

Reflection and Refraction

- Snell’s Law

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}
\]
Light incident to a less dense medium (i.e., \( n_1 > n_2 \))

Critical angle (i.e., when \( \theta_2 = 90^\circ \))

\[
\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)
\]

For glass/air interface, \( n_1 = 1.5 \) and \( n_2 = 1 \), then \( \theta_c = 41.8^\circ \)

- \( \theta_1 < \theta_c \)
- \( \theta_1 = \theta_c \)
- \( \theta_1 > \theta_c \)

Total Internal Reflection

Optical Fiber

- Core has greater refractive index for total internal reflection
- Size varies
  - Large cores for imaging and high-power illumination
  - Small cores for optical communications
  - Standard fibers for telecom: 125\(\mu\)m cladding/9\(\mu\)m core

Step Index Fiber

\( n_1 > n_2 \)
Optical Fiber

\[ \phi_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]

\[ \theta_c = 90^\circ - \phi_c \]

\[ n \sin \theta_{\text{max}} = n_1 \sin \theta_c = n_1 \cos \phi_c = \sqrt{n_1^2 - n_2^2} \]

Maximum acceptance angle: \[ \theta_{\text{max}} = \sin^{-1} \left( n_1^2 - n_2^2 \right)^{1/2} \]

Numerical Aperture: \[ NA = \sin \theta_{\text{max}} = \left( n_1^2 - n_2^2 \right)^{1/2} \]

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Principle of Linear Superposition

Two plane waves of the same frequency \( \omega \)

\[ \vec{E}_1 = \vec{E}_1 \exp \left[ i \left( \vec{k}_1 \cdot \vec{r} - \omega t + \phi_1 \right) \right] \]

\[ \vec{E}_2 = \vec{E}_2 \exp \left[ i \left( \vec{k}_2 \cdot \vec{r} - \omega t + \phi_2 \right) \right] \]

Irradiance:

\[ I = |\vec{E}|^2 = \vec{E} \cdot \vec{E}^* = |\vec{E}_1|^2 + |\vec{E}_2|^2 + 2 \vec{E}_1 \cdot \vec{E}_2 \cos \theta \]

\[ = I_1^2 + I_2^2 + 2 \vec{E}_1 \cdot \vec{E}_2 \cos \theta \]

\[ \theta = \vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r} + \phi_1 - \phi_2 \]

- \( \phi_1, \phi_2 \) is constant \( \rightarrow \) mutually coherent
- \( \phi_1, \phi_2 \) is randomly changing with time \( \rightarrow \) mutually incoherent \( \rightarrow \) no fringes
- The interference term depends on polarization. If the polarizations are mutually orthogonal, then \( \vec{E}_1 \cdot \vec{E}_2 = 0 \). No interference fringes.
Theory of Partial Coherence

Two plane waves of the same frequency \( \omega \) but with varying phases

\[
I = \langle E \cdot E^* \rangle = \left\langle \left( \bar{E}_1 + \bar{E}_2 \right) \left( \bar{E}_1^* + \bar{E}_2^* \right) \right\rangle \\
= \left\langle |\bar{E}_1|^2 + |\bar{E}_2|^2 + 2 \Re \left( \bar{E}_1^* \bar{E}_2 \right) \right\rangle \\
= I_1^2 + I_2^2 + 2 \Re \left( E_1 E_2^* \right)
\]

Time average

\[
\langle f \rangle \triangleq \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t) dt
\]

- Usually \( E_1 \) and \( E_2 \) originate from a common source. They differ due to traveling different optical paths.

Theory of Partial Coherence

Mutual Coherence Function (or Correlation Function)

\[
\Gamma_{12}(\tau) = \left\langle E_1(t) E_2^*(t + \tau) \right\rangle
\]

Self-Coherence Function (or Autocorrelation Function)

\[
\Gamma_{11}(\tau) = \left\langle E_1(t) E_1^*(t + \tau) \right\rangle
\]

Degree of Partial Coherence

\[
\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0) \Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}
\]

\[
|\gamma_{12}| = 1 \quad \text{(complete coherence)} \\
0 < |\gamma_{12}| < 1 \quad \text{(partial coherence)} \\
|\gamma_{12}| = 0 \quad \text{(complete incoherence)}
\]
Theory of Partial Coherence

Fringes

\[ I = I_1 + I_2 + 2\sqrt{I_1 I_2} \text{Re} \gamma_{12} (\tau) \]

\[ I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1 I_2} |\gamma_{12}| \]

\[ I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1 I_2} |\gamma_{12}| \]

Fringe Visibility

\[ V \equiv \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{2\sqrt{I_1 I_2} |\gamma_{12}|}{I_1 + I_2} \]

\[ V = |\gamma_{12}| \quad \text{if } I_1 = I_2 \]

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Theory of Partial Coherence

Coherence Time \( \tau_0 \) and Coherence Length \( l_c \)

\[ l_c = c\tau_0 \]

For a monochromatic wave with a finite duration \( \tau_0 \), i.e.,

\[ f(t) = \begin{cases} e^{-i\omega_0 t} & \text{for } -\frac{\tau_0}{2} < t < \frac{\tau_0}{2} \\ 0 & \text{otherwise} \end{cases} \]

Taking Fourier transform yields

\[ g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\tau_0/2}^{\tau_0/2} e^{-i\omega t} dt = \frac{1}{\sqrt{2\pi}} \frac{\sin[(\omega - \omega_0)\tau_0 / 2]}{\omega - \omega_0} \]
Theory of Partial Coherence

Coherence Time $\tau_0$ and Coherence Length $l_c$

Power spectrum:

$$G(\omega) = |g(\omega)|^2 = \frac{2}{\pi} \sin^2 \left[ \left( \omega - \omega_0 \right) \frac{\tau_0}{2} \right] \left( \omega - \omega_0 \right)^2$$

$\Rightarrow$ The first minima occur at $\omega = \omega_0 \pm \frac{2\pi}{\tau_0}$

$\Rightarrow$ Spectral width $\Delta \omega = \frac{2\pi}{\tau_0}$

$\Rightarrow$ $\Delta v = \frac{1}{\tau_0}$

$\Rightarrow$ $l_c = c \langle \tau_0 \rangle = \frac{c}{\Delta v} = \frac{\lambda^2}{\Delta \lambda}$

Doppler Effect

Observed frequency $f'$

$$f' = f \left( \frac{c}{c+v} \right) = f \left( 1 - \frac{v}{c} + \frac{v^2}{c^2} - \cdots \right) \approx f \left( 1 - \frac{v}{c} \right)$$
Lenses

Spherical lens, cylindrical lens, GRIN (gradient-index) rod lens

Numerical Aperture

\[ NA = n \sin \alpha \]
\[ NA \approx \frac{D}{2f} \quad \text{For small NA} \]

F-number

\[ f \# = \frac{f}{D} \]

Resolution

\[ r = \frac{0.61\lambda}{NA} \]

Gaussian Beam

Beam waist (or spot size)

\[ 2w_0 = 2.44\frac{\lambda f}{D} \]

Raleigh Range

\[ z_R = \frac{\pi w_0^2}{\lambda} \]

Beam radius at z

\[ w(z) = w_0 \left[ 1 + \left( \frac{z}{z_R} \right)^2 \right]^{-1/2} \]

Divergence angle

\[ \theta \approx \frac{w(z)}{z} = \frac{\lambda}{\pi w_0} = \frac{w_0}{z_R} \]
Polarization

- If the electric field is oscillating along a straight line, it is called a linearly polarized (LP) or plane polarized wave.
- If the $E$ field rotates in a circle (constant magnitude) or in an ellipse then it is called a circular or elliptically polarized wave.
- Natural light has random polarization.

Adding two linearly polarized waves with zero phase shift will generate another linearly polarized wave.
Adding two linearly polarized waves with a phase shift will produce an **elliptically polarized light**

Adding two linearly polarized waves with equal amplitude and 90° phase shift results in a **circular polarized wave**
We discussed some basic optical phenomena: reflection, refraction, polarization, interference, diffraction, coherence and Doppler shift.

- Lenses: numerical aperture
- Gaussian beam
- Optic Fibers:
  - Total reflection
  - Acceptance angle

References: