Lecture 14

Agenda:
- Mechanical Modeling of RF MEMS Devices
  - Static Analysis

Most figures and data in this lecture, unless cited otherwise, were taken from RF MEMS Theory, Design and Technology by G. Rebeiz, 2003.

### Different Configurations of RF MEMS Devices

<table>
<thead>
<tr>
<th>Actuation Mechanisms</th>
<th>Voltage (V)</th>
<th>Current (mA)</th>
<th>Power (mW)</th>
<th>Size</th>
<th>Switching time (μs)</th>
<th>Contact force (μN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrostatic</td>
<td>20-80</td>
<td>0</td>
<td>0</td>
<td>Small</td>
<td>1-200</td>
<td>50-1000</td>
</tr>
<tr>
<td>Thermal</td>
<td>3-5</td>
<td>5-100</td>
<td>0-200</td>
<td>Large</td>
<td>300-10,000</td>
<td>500-4000</td>
</tr>
<tr>
<td>Magnetic</td>
<td>3-5</td>
<td>20-150</td>
<td>0-100</td>
<td>Medium</td>
<td>300-1,000</td>
<td>50-200</td>
</tr>
<tr>
<td>Piezoelectric</td>
<td>3-20</td>
<td>0</td>
<td>0</td>
<td>Medium</td>
<td>50-500</td>
<td>50-200</td>
</tr>
</tbody>
</table>

### Contact Type (Switches Only)

- Metal-to-metal: DC – 60 GHz
- Capacitive: 10 – 120 GHz

### Static Analysis

- Electrostatic actuation
- Pull-in voltage
- DC Hold-Down Voltage
- Self-actuation of MEMS capacitive switches
- Three-plate electrostatic design
- Stabilization of electrostatic actuated beams
- Temperature effect
- Effect of acceleration and acoustic forces
- Effect of Holes in a Beam

### RF MEMS Switches

- **Series Switch**
  - [Diagram of Series Switch]
  - Circuit diagram

- **Shunt Switch**
  - [Diagram of Shunt Switch]
  - Circuit diagram
Electrostatic actuation

### Fixed-Fixed Beam

\[ F = k(x) = 32Ew \left( \frac{t}{l} \right)^3 \]

### Cantilever Beam

\[ k = \frac{3EI}{l^3} = \frac{1}{4} Ew \left( \frac{t}{l} \right)^3 \]

**Stretching Effect** (only for fixed-fixed beams)

Nonlinear stretching restoring force:

\[ F = k_s (g_0 - g) = \frac{\pi^2 Ewt}{8g^3} (g_0 - g)^3 \]

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**DC Hold-Down Voltage**

\[ C = \frac{\varepsilon_0 A}{g + t_s / \varepsilon_r} \]

\[ F_e = V^2 \frac{\varepsilon_0 \varepsilon_r A}{2 \left( g + t_s / \varepsilon_r \right)^2} \]

\[ \gamma = \begin{cases} 1 & (g \neq 0) \\ 0.4 - 0.8 & (g = 0) \end{cases} \]

(Capacitance reduction due to interface roughness)

\[ F_s = F_e = k_s (g_0 - g) + k_s (g_0 - g)^3 \]

**Hold-Down Voltage**

\[ V_h = \sqrt{\frac{2k_s}{\varepsilon_0 \varepsilon_r A}} (g_0 - g) \left( g + \left( \frac{t_s}{\varepsilon_r} \right) \right) \]

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**Self-Actuation**

**Shunt Capacitive Switch**

\[ V_{sw} \sim \frac{V}{Z_0} \]

**Series Capacitive Switch**

\[ V_{sw} \sim \frac{V}{C_w C_d / Z_0} \]

\[ V_{sw} \]: Switch voltage; \( C_w \): up-state capacitance; \( C_d \): down-state capacitance

**Rectifying Effect** due to \( V^2 \) dependence of electrostatic force
Self-Actuation

Typically the frequency of the RF signal is much higher than the resonant frequency of the MEMS switch. But the DC offset of the RF signal is applied to the switch. For high-power RF signals, this DC offset may generate large actuation force, so called Self-Actuation.

\[ V_+ = V_{pk} \sin(o t) = \sqrt{2PZ_0} \sin(o t) \]

\( V_{pk} \): voltage amplitude of RF signal; \( Z_0 \): characteristic impedance of t-line; \( o \): RF frequency

For a shunt switch, it is assumed that \( C_d \) is small and thus the reflection coefficient is small.

\[ V_{sw} = V_+ + V_- \approx V_+ \]

\[ V_{sw}^2 \approx (V_+)^2 = \frac{1}{2} V_{pk}^2 \left( 1 - 2 \cos(2o t) \right) = V_{dc-eq}^2 \left( 1 - 2 \cos(2o t) \right) \]

\[ F_s = \frac{1}{2} \frac{\varepsilon_0 A}{g^2} V_{sw}^2 = \frac{1}{2} \frac{\varepsilon_0 A}{g^2} V_{dc-eq}^2 \left( 1 - 2 \cos(2o t) \right) = \frac{1}{2} \frac{\varepsilon_0 A}{g^2} V_{dc-eq}^2 \]

RF Hold-Down Voltage

Similar to the self-actuation, the DC offset of RF signals may generate electrostatic force large enough to hold the switch down even when the switch voltage is already turned off.

**Shunt Switch**

\[ V_{sw} = \frac{2V^+}{\omega C_d Z_0} = \frac{2\sqrt{2PZ_0}}{\omega C_d Z_0} \text{ for } \omega C_d Z_0 \gg 1 \]

Also \( V_{dc-eq} = V_{sw} / \sqrt{2} \)

\[ P_{shunt} = \frac{V_{dc-eq}^2}{4Z_0} \left( \omega C_d Z_0 \right)^2 = \frac{V_{dc-eq}^2}{4Z_0} \left( Z_0 / Z_{C_d} \right)^2 \]

For example, \( V_h = 5V, C_d = 2pF, Z_0 = 50\Omega \)

\[ P_{shunt} = 5W \]

**Series Switch**

\[ V_{sw} = \frac{V^+}{\omega C_d Z_0} = \frac{\sqrt{2PZ_0}}{\omega C_d Z_0} \text{ for } \omega C_d Z_0 \gg 1 \]

Also \( V_{dc-eq} = V_{sw} / \sqrt{2} \)

\[ P_{series} = \frac{V_{dc-eq}^2}{Z_0} \left( \omega C_d Z_0 \right)^2 = \frac{V_{dc-eq}^2}{Z_0} \left( Z_0 / Z_{C_d} \right)^2 \]

For example, \( V_h = 5V, C_d = 2pF, Z_0 = 50\Omega \)

\[ P_{series} = 20W \]

Not a big concern since most RF MEMS switches can only handle up to 1W due to other limitations.
Due to the instability after 2/3 of the gap, the maximum controllable capacitance ratio is 1.5.

In practice, the achievable capacitance ratio using fixed-fixed beams is 1.2-1.4 because of curling and parasitic capacitance.

Three-plate Design for Higher Capacitance Ratio

Middle plate can move 1/3 on both directions

\[
C_{\text{max}} = \frac{\varepsilon_r A}{g - g/3} = \frac{3}{2} \frac{\varepsilon_r A}{g}
\]

\[
C_{\text{min}} = \frac{\varepsilon_r A}{g + g/3} = \frac{3}{4} \frac{\varepsilon_r A}{g}
\]

Maximum capacitance ratio = 2.0

Instability control by placing a series capacitor in series with a MEMS bridge capacitor

\[
V_b = \frac{V_s}{1 + C_b / C_s}
\]

\[
V_s - V_b = \frac{V_s}{1 + C_b / C_s}
\]

\[V_s\] increases with \[V_b\]. Meanwhile, \[C_b\] increases with \[V_b\] as the electrostatic force pulls the bridge down. Thus, more voltage will drop across \(C_s\) instead of \(C_b\), resulting in a negative feedback. The new position of the instability is given by

\[
g_p = \frac{g_s}{3} (2 - K)
\]

where \(K = C_{1b} / C_s\).

When \(K=2\), the instability is completely eliminated.

When large fringing capacitance exists, the instability position will be shifted to

\[
V_p = \frac{8k_c}{27 \varepsilon_r A} (1 + K)^{1/2}
\]

The stabilization is obtained at the price of increased pull-in voltage. For \(K=2\), the pull-down voltage is increased by a factor of 5.2.

When large fringing capacitance exists, the instability position will be shifted to

\[
g_p = \frac{g_s}{3} \left( 2 - \frac{K}{1 + C_b / C_s} \right)
\]

Thus, to completely eliminate the instability,

\[C_s = \frac{C_{1b}}{2} - C_f\]
Effect of Temperature Variation

Fixed-fixed beams
- Residual stress
- Stress gradient
  - Spring constant change → Pull-down voltage change

Cantilever beams
- Stress gradient
- Bi-layer structures
- Thermal expansion coefficient difference → Curling or bending

Solutions:
- Temperature stabilization
- Temperature compensation
- Single-crystal silicon
- Highly stressed thin-film layer

Effect of Acceleration and Acoustic Forces

Acceleration
\[ \Delta x = \frac{F}{k} = \frac{m}{k} \frac{a}{a} \]

For a MEMS switch whose resonant frequency is 50 kHz, a 1µm-displacement requires an acceleration of 10,000g.

Acoustic waves
\[ \Delta x = \frac{F}{k} = \frac{PA}{k} \]

For a MEMS switch with k=10 N/m and an area of 100 x 100 µm², a pressure wave of 1 Pa (equivalent to 94dB SPL) will produce only 1 nm deflection. To close a 1µm gap, it will require 154 dB SPL.

SPL: sound pressure level. 0 dB SPL is the threshold of hearing
34 dB SPL: Library 63 dB SPL: Office
83 dB SPL: Street traffic 90 dB SPL: Heavy Truck

Effect of Holes in the Beam

Ligament efficiency
\[ \mu = \frac{l_p}{l} \]

Residual stress reduction
\[ \Delta \sigma = \left(1 - \mu\right) \sigma_0 \]
where \( \sigma_0 \) is residual stress with no holes.

Capacitance
The effect of the holes on the up-state capacitance will be negligible if \( d > 3g_0 \). Thus, the electrostatic force will not be affected by the hole density or pattern as long as \( d > 3g_0 \).

The down-state capacitance is reduced, resulting in a reduced capacitance ratio.

Summary
- Capacitive switches are most popular
- There are shunt and series capacitive switches
- Gap “pull-in” is good for binary switches but limits capacitance ratio to less than 1.5 for analog-mode switches
- The instability position at “pull-in” can be extended by a series capacitor
- DC hold-down voltage is typically much smaller than pull-down voltage
- The DC term in the electrostatic force generated by a RF signal may lead to self actuation or self hold-down of MEMS capacitive switches
- The RF self-actuation and hold-down set limits on the maximum RF handling power to capacitive switches
- Three-plate electrostatic design can increase capacitance ratio to 2.0
- Temperature stability is still a big challenge
- Effects of acceleration and acoustic forces are negligible
- Holes in a beam will reduce residual stress, stiffness, down-state capacitance and capacitance ratio but have little effect on up-state capacitance