Lecture 15
RF MEMS (3)

- Agenda:
  - S-Parameters
  - Mechanical Modeling of RF MEMS Devices
    - Dynamic Analysis
  - Electromagnetic Modeling

Most figures and data in this lecture, unless cited otherwise, were taken from RF MEMS Theory, Design and Technology by G. Rebeiz.

S-Parameters

- Y and Z parameter models use input and output voltage and current signals.
- Scattering Parameters, or S-parameters have inputs and outputs expressed in power.
- $a$ is assigned to incident values while $b$ indicates reflected values.
- S-parameters are transmission and reflection coefficients.
- Transmission coefficients are referred to as gains and attenuations.
- Reflection coefficients are related to voltage standing wave ratios (VSWRs) and impedances.
S-Parameters

\[ S_{11} = \frac{b_1}{a_1} \mid a_2 = 0 \]

= Input reflection coefficient with the output port terminated by a matched load \((Z_L = Z_0)\) sets \(a_2 = 0\).

\[ S_{22} = \frac{b_2}{a_2} \mid a_1 = 0 \]

= Output reflection coefficient with the input terminated by a matched load \((Z_S = Z_0)\) sets \(V_S = 0\).

\[ S_{21} = \frac{b_2}{a_1} \mid a_2 = 0 \]

= Forward transmission (insertion) gain with the output port terminated in a matched load.

\[ S_{12} = \frac{b_1}{a_2} \mid a_1 = 0 \]

= Reverse transmission (insertion) gain with the input port terminated in a matched load.

Reflection coefficient \(\Gamma = S_{11} \rightarrow\) Smith Chart
Dynamic Analysis of MEMS Switches

- Damping
- Switching time
- Release time

Damping

Gas Fundamentals

Mean-free path

$$\lambda = \frac{1}{\sqrt{2\pi N\sigma^2}}$$

N is the number density of gas. \(\sigma\): gas molecule’s diameter.

\(\lambda_0=0.07-0.09\,\mu m\) for most gases at STP (standard temperature and pressure, i.e., \(P_0=101\,\text{kPa}\) at room temperature).

For a gas at a pressure \(P_a\), the mean free path can be obtained by

$$\lambda_a = \frac{P_a}{P_0} \lambda_0$$

For example, if \(P_a = 1\,\text{morr} = 7.5\,\text{Pa}\), then \(\lambda_a = 1\,\text{mm}\), which is much greater than the gaps of MEMS devices.
Damping

- **Gas Fundamentals**

  Knudsen Number \[ K_n = \frac{\lambda}{g} \] where \( g \): air gap.

  Knudsen number is a measure of the viscosity of the gas. The smaller the Knudsen number, the more viscous the gas or fluid is.

  Coefficient of viscosity

  \[ \mu = 1.2566 \times 10^{-6} \sqrt{T} \left(1 + \frac{110.33}{T}\right)^{-1} \text{ kg/m} \cdot \text{s} \]

  \( T \) is in Kelvin and this equation is for ideal and quasi-ideal gases. At STP, \( \mu = 1.845 \times 10^{-5} \text{ kg/(m} \cdot \text{s)}. \)

  Viscosity changes with Knudsen number. Veijola et al derived the dependence:

  \[ \mu_e = \frac{\mu}{1 + 9.638K_n^{1.159}} \]

- **Gas Fundamentals**

  Squeeze Number

  \[ \sigma(\omega) = \frac{12 \mu_e l^2}{P_g g^2} \omega \]

  where \( \omega \) is the applied mechanical frequency.

  \( l \): the characteristic length. For a circular membrane, \( l \) is the radius. For a rectangle, \( l \) is the shortest dimension.

  Squeeze-film damping \( \rightarrow b \)

  Gas compression \( \rightarrow \) spring force \( \rightarrow k_s \)

  At low frequencies \( k_s = 0 \).

  \[ b = \frac{3}{2\pi} \frac{\mu A^2}{g^3} \]

  At high frequencies (i.e., high squeeze number), \( k_s \rightarrow P_g A / g \)

  High squeeze number will increase the effective stiffness and thus the resonance frequency.
Damping

- **Quality Factor at Atmospheric Pressure**
  
  Cantilever Beam
  
  \[ Q_{\text{cant}} = \frac{\sqrt{E\mu l^2}}{\mu (wl)^2} g_0^3 \]
  
  where \( w \) and \( l \) are the width and length of the cantilever.

  Fixed-fixed beam
  
  \[ Q_f = \frac{\sqrt{E\mu l^2}}{\mu (wl/2)^2} g_0^3 \]
  
  Given \( l=300\mu m, w=60\mu m, t=1\mu m \) and \( g_0=3\mu m \) \( \Rightarrow Q=1.0 \)

  At very low pressures, \( \mu \to 0 \). The damping coefficient is limited by anchor loss, internal friction loss and thermoelastic dissipation (TED).

- **Damping Variation Versus Gap Height**
  
  \[ Q = Q \left(1 - \frac{x}{g_0}\right)^{3/2} \left(1 + 9.638K_1^{1.159}\right) \]
  
  \( Q \) is small-displacement quality factor at \( g=g_0 \). Derived by Sadd and Siffier (Trans. ASME, pp.1366-1370, Nov. 1975)

Nonlinear Dynamic Analysis

- **Switching time**
  
  Equation of Motion
  
  \[ m\ddot{x} + b\dot{x} + kx + k_x x^3 = F_v + F_c \]
  
  where
  
  \[ F_v = \frac{1}{2} \left( \frac{e_0 AV^2}{g_0 + t / \kappa - x} \right) \]
  
  \[ V = V_i \left( \frac{C_1 dV}{dt} + V \frac{dC}{dt} \right) \]
  
  \[ F_c = \frac{C_1 A}{(g_0 - x)^3} - \frac{C_2 A}{(g_0 - x)^{10}} \]

  Attractive van der Waals force
  
  Repulsive nuclear contact force

  \( C_1 \) and \( C_2 \) strongly depends on surface materials and conditions. In this analysis, \( C_1=10^{-80}Nm; C_2=10^{-75}Nm^8 \)

  Chan et al, IEEE MTT-S Digest, 1997
### Nonlinear Dynamic Analysis

#### Parameters of the MEMS Beam for this analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( l )</td>
<td>300 ( \mu \text{m} )</td>
</tr>
<tr>
<td>Thickness</td>
<td>( t )</td>
<td>0.8 ( \mu \text{m} )</td>
</tr>
<tr>
<td>Pull-down electrode length</td>
<td>( W )</td>
<td>100 ( \mu \text{m} )</td>
</tr>
<tr>
<td>Bridge width</td>
<td>( w )</td>
<td>100 ( \mu \text{m} )</td>
</tr>
<tr>
<td>Gap height</td>
<td>( g_0 )</td>
<td>3 ( \mu \text{m} )</td>
</tr>
<tr>
<td>Spring constant (Au and Al)</td>
<td>( k )</td>
<td>10 N/m</td>
</tr>
<tr>
<td>Residual stress (Au)</td>
<td>( \sigma )</td>
<td>9.5 MPa</td>
</tr>
<tr>
<td>( k ) Components (Au)</td>
<td>( k', k'' )</td>
<td>(( k' = 2.67 ) N/m, ( k'' = 7.33 ) N/m)</td>
</tr>
<tr>
<td>Density of Al</td>
<td>( \rho_{\text{Al}} )</td>
<td>2,700 kg/m(^3)</td>
</tr>
<tr>
<td>Density of Au</td>
<td>( \rho_{\text{Au}} )</td>
<td>19,320 kg/m(^3)</td>
</tr>
<tr>
<td>Effective mass</td>
<td>( m_e )</td>
<td>0.35/(( l w t ))p</td>
</tr>
<tr>
<td>Mechanical resonance frequency (Al)</td>
<td>( f_{0,\text{Al}} )</td>
<td>106 kHz</td>
</tr>
<tr>
<td>Mechanical resonance frequency (Au)</td>
<td>( f_{0,\text{Au}} )</td>
<td>39.5 kHz</td>
</tr>
</tbody>
</table>

#### Switching Time

- **Switching time of fixed-fixed beams**
  
  Matlab/Simulink can be used to obtain the transient response

  - Resonance frequencies:
    - Al beam: 106 kHz
    - Au beam: 39.5 kHz
  
  - Applied voltage:
    - \( V_s = 1.4 \) \( V_p \)
  
  - Constant voltage
  - Displacement-dependent damping factor
  - Higher Q makes switching faster, but little effect above Q=2
Switching Time

- Switching time varying with applied voltage

- Au beam; Resonance frequency: \( f_0 = 39.5 \text{ kHz} \)
- Each cycle: 25 \( \mu \text{s} \); \( Q = 1 \)

- For \( Q > 2 \) (Acceleration-limited), switch time \( t_s \approx 0.58 \frac{(V_p/V_s)}{f_0} \)
- For \( Q < 0.5 \) (Damping-limited), \( t_s \approx 0.36-1.0 \frac{(V_p/V_s)^2}{(Qf_0)} \)

Release Time

- Release response can be obtained by setting \( F_e = 0 \) in the nonlinear dynamic equation.
- The bridge oscillates if \( Q > 1 \)
- For best release response, \( Q \approx 1 \)
- Variable damping has greater effect

- Au beam
- Resonance frequency: \( f_0 = 39.5 \text{ kHz} \)
- Each cycle: 25 \( \mu \text{s} \)
Capacitive Shunt Switches

- Circuit model
- Current distribution
- Series Resistance
- Loss

Top view

Cross-sectional View

- G: 50-100 μm
- W: 50-100 μm
- L: 250-400 μm
- w: 25-180 μm
- g: 1.5-5 μm
Capacitive Shunt Switches

- **Current Distribution**
  - **Up-state Position**
  - **Down-state Position**

- No RF current on the middle portion of the bridge
- RF current is carried only at the edge of the t-line
- RF current is concentrated on the bridge edge over the CPW gap.
- Changing the bridge width does not affect the current distribution.
- For down-state position, only one edge of the bridge carries the current. This edge is a short circuit to the incoming wave.

**Circuit Model**

\[ Z_s = R_s + j\omega L + \frac{1}{j\omega C} \begin{cases} 
1 / j\omega C & \text{for } f \ll f_0 \\
R_s & \text{for } f = f_0 \\
\omega L & \text{for } f \gg f_0 
\end{cases} \]

\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \]

- \( C = C_u \) or \( C_d \) depending on the switch position.
- Typically, \( f_{u,d} \sim 300\text{GHz} \) and \( f_{d,d} \sim 50\text{GHz} \).
- For up-state position, the switch can be modeled as a shunt capacitance to ground.
- The inductance plays important role for the down-state position.
Capacitive Shunt Switches

- **Circuit Model**

Series Resistance

\[ R_{s2}: 0.01 \sim 0.1 \Omega \]

\[ \alpha: t\text{-line loss, } 0.2 \text{-}2 \text{dB/cm (or } 0.17 \sim 1.7 \text{ Np/m)} \]

- If the bridge thickness is smaller than two skin depths, the switch resistance is constant with frequency.
- For thick MEMS bridges, the switch resistance changes as \( (f)^{1/2} \) with frequency.

Inductance

\[ L: 1 \sim 100 \text{ pH} \]

Switches with meander suspensions for small spring constants have large inductance.

Simulators: IE3D, Sonnet, HFSS

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Capacitive Shunt Switches

- **Loss**

\[ \text{Loss} = 1 - |S_{11}|^2 - |S_{21}|^2 \]

\[ \text{Loss} = \frac{\text{Power loss in MEMS Bridge}}{\text{Power incident on MEMS switch}} = \frac{I^2 R_s}{\left| V^1 \right|^2 / Z_0} \]

\[ \text{Loss}(dB) = 10 \log (1 - \text{Loss}) \]
**Capacitive Shunt Switches**

- **Loss**
  
  **Up-state Position:** $Z_s >> Z_0$
  
  $$\text{Loss} = \frac{R_L Z_0}{|Z_s|^2} = \omega^2 C_n^2 R_L Z_0$$
  
  Total Loss: $\text{Loss}_u (dB) = \alpha l (dB) + \omega^2 C_n^2 R_L Z_0 (dB)$ (~0.05 dB)
  
  **Down-state Position:** $Z_s << Z_0$
  
  $$\text{Loss} = \frac{4R_L Z_0}{|Z_0|^2} \approx \frac{4R_L}{Z_0}$$
  
  Total Loss: $\text{Loss}_d (dB) = \alpha l (dB) + \frac{4R_L}{Z_0} (dB)$ (~0.15 dB)

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**Summary**

- **Introduction to S-Parameters**
- **Mean-free path at STP:** 0.07 µm
- **Knudsen number < 0.1 → Viscous**
- **Switching time**
  - Nonlinear equation of motion
  - Matlab/Simulink can be used to obtain transient response
  - Increasing drive voltages decreases switching time
  - Higher Q → faster switching
- **Release time**
  - High Q leads to oscillation
  - Optimal Q $\approx 1$