Introduction to Fluidics

**Today:**
- Surface Tension
- Squeeze-Film Damping
- Electrokinetic Effects
  - Electroosmotic flow
  - Electrophoresis

📖 Reading: Senturia, Chapter 13, p.332-349
Surface Tension

**Physics**

- Molecules at surface experience different forces than moles in bulk and are energetically at higher energy.
- Distributed force called ‘surface tension’ tends to minimize free surface and attain energy minimum.
  - Suspended liquid tends to form spherical drop (lowest surface/volume ratio).
Surface Tension

- **Physics**
  - Surface tension, \( T = [N/m] = [J/m^2] \)
    
    \[
    \frac{\text{Surface energy}}{\text{volume}} = \frac{(4\pi r^2)T}{\left(\frac{4\pi r^3}{3}\right)} = \frac{3T}{r}
    \]
    
    - smaller bubbles are energetically less favorable
    - large forces associated with formation and collapse (cavitation) of bubbles

- **Capillary force**
  - contact angle, \( \theta \)
  - \( \theta \) depends on liquid, surface roughness, contamination, roughness, etc.

At equilibrium, total force on fluid in capillary is zero.

\[
F_{\text{surface tension}} = F_{\text{gravitational}}
\]

\[
T(2\pi r) \cos \theta = \rho (\pi r^2 h) g \quad \Rightarrow \quad h = \frac{2T \cos \theta}{\rho g r}
\]

Ref. Senturia, Microsystem Design, p.321
MEMS:

Surface tension force comparable to applied pressure force

<table>
<thead>
<tr>
<th>Liquid (@ 20°C)</th>
<th>Surface Tension, $\sigma$ (mN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>28.9</td>
</tr>
<tr>
<td>Carbon tetrachloride</td>
<td>27.0</td>
</tr>
<tr>
<td>Glycerin</td>
<td>63.0</td>
</tr>
<tr>
<td>Hexane</td>
<td>18.4</td>
</tr>
<tr>
<td>Kerosine</td>
<td>26.8</td>
</tr>
<tr>
<td>Lube oil</td>
<td>25–35</td>
</tr>
<tr>
<td>Mercury</td>
<td>484</td>
</tr>
<tr>
<td>Methanol</td>
<td>22.6</td>
</tr>
<tr>
<td>Octane</td>
<td>21.8</td>
</tr>
<tr>
<td>Water</td>
<td>72.8</td>
</tr>
</tbody>
</table>

(a) In contact with air

Example:

Water in 20 $\mu$m dia. channel
Assume $\theta=30^\circ$ (depends on material and surface conditions)

$$(2\pi r)T\cos\theta=\Delta p(\pi r^2)$$

$\Delta p = 12.5 kPa \approx 1/8 atm$

Ref. Fox and McDonald, Introduction to Fluid Mechanics, p. 761.
Examples:

- **Walking on water**
  - Small insects such as the water strider can walk on water because their weight is not enough to penetrate the surface.

- **Floating a needle**
  - If carefully placed on the surface, a small needle can be made to float on the surface of water even though it is several times as dense as water. If the surface is agitated to break up the surface tension, then needle will quickly sink.
Dissipation mechanism

- Motion of upper plate relative to fixed bottom plate with viscous fluid between plates
  - Decreasing gap leads to:
    - increase in pressure between plates
    - flow tangential to plates (squeeze flow)
  - Viscous drag during flow creates dissipative force on plate opposing motion
  - Dissipative force is termed ‘squeeze-film damping’

Important second-order effect in MEMS devices including microphones and comb drive-based devices
Assumptions

- Fluid is gas that obeys ideal gas law.
- Gap, $h$, is much smaller than lateral dimensions of plate.
- The motion is sufficiently slow such that the inertial term may be neglected compared to the viscous term (known as Stokes flow.)
- No pressure gradient transverse to the plate.
- Lateral flow has Poiseuille-like velocity profile.
- System is isothermal, i.e. any temperature rise is quickly compensated by transfer to the walls.

Under these assumptions, combining the conservation of mass, Navier-Stokes equation and ideal gas law will give the **Reynolds equation**:

\[
12\eta \frac{\partial (Ph)}{\partial t} = \nabla \cdot \left[ (1 + 6K_n) h^3 P \nabla P \right]
\]

where \( P(x,y) \) does not depend on \( z \). \( K_n \), or **Knudsen number**, is the ratio of the mean free path to the gap dimension. If \( K_n << 0.1 \), the no-slip boundary condition can be applied.

For small-amplitude motion we may linearize the Reynolds equation around \( h_0 \).
Solution in Laplace domain gives the squeeze-film damping force on the plate:

\[ F(s) = \frac{96\eta LW^3}{\pi^4 h_0^3} \sum_{n \text{ odd}} \frac{1}{n^4 h_0^4} \frac{1}{1 + \frac{s}{\alpha_n}} sZ(s) \]

where \( Z(s) \) is the Laplace transform of the source displacement.

Approximating using the first term gives:

\[ F(s) = \frac{b}{1 + \frac{s}{\omega_c}} sZ(s) \]

where \( b = \frac{96\eta LW^3}{\pi^4 h_0^3} \)

\( \omega_c = \frac{\pi^2 h_0^2 P_0}{12\eta W^2} \)

\( \sigma_d = \frac{12\eta W^2}{h_0^2 P_0} \omega = \pi^2 \frac{\omega}{\omega_c} \)

Ref. Senturia, Microsystem Design, p.338
Electrokinetic Effects

- Ionic Double Layers
  - Helmholtz Planes
  - Diffuse Double Layer
- Electroosmotic Flow
- Electrophoresis
- Mixing
- Diffusion Effects
- Pressure Effects
Ionic Double Layers

**Compact layers:**
- Inner and outer Helmholtz planes
- Electrolyte/solid interface
- Outer layer has opposite charge from the inner layer.
- Charge densities in the two layers change with the voltage between the metal and the electrolyte
- Not charge-balanced.

**Diffuse double layer**
- Balance the net charge in the two Helmholtz planes.
- The potential near the wall is given by

\[ \hat{\phi} = \phi_w e \frac{z}{L_d} \]

- \( L_d \): Debye length. 1um in pure water, 0.3um in a 1ml solution.
- \( \phi_w \): wall potential at the outer edge of the compact layer
Ionic Double Layers

From Poisson equation, the charge density in the diffuse layer is given by

\[ \rho_e = -\varepsilon \nabla^2 \phi = -\frac{\varepsilon}{L_D^2} \phi \]

Then the total charge per unit area is

\[ \sigma_d = \int_0^\infty \rho_e \, dz = -\frac{\varepsilon \phi_w}{L_D} \]

Thus,

\[ \phi_w = -\frac{\sigma_d L_D}{\varepsilon} = \frac{\sigma_w L_D}{\varepsilon} \]

Finally, we have

\[ \hat{\phi} = \frac{\sigma_w L_D}{\varepsilon} e^{-\frac{z}{L_d}} \]
Electroosmotic Flow

- Electrolyte bounded by insulating walls.
- There is a tangential electric field.

For incompressible liquid, the Navier-Stokes reduces to:

\[ \rho_m \frac{D U_x}{Dt} = - \frac{dP}{dx} + \eta \nabla^2 U_x + \rho_e E_x \]

where \( \rho_e = - \frac{\sigma_w}{L_D} e^{\frac{-z}{L_D}} \)

Assume steady state, fully developed flow. Then

\[ \frac{d^2 U_x}{dz^2} = \frac{\sigma_w E_x}{\eta L_D} e^{\frac{-z}{L_D}} \]
No-slip condition yields

\[ U_x(z) = \frac{\sigma_w E_x L_D}{\eta} \left(1 - e^{-\frac{z}{L_D}}\right) \]

Maximum flow velocity occurs at the center, and is equal to

\[ U_0 = \frac{\sigma_w E_x L_D}{\eta} \]

- Highly sheared boundary layer within 3 Debye lengths of the walls
- Uniform velocity in most of the sample volume  
  \( \rightarrow \) Plug Flow
- Flow velocity is proportional to the wall charge \( \sigma_w \)  
  \( \rightarrow \) Charge control

Ref. Senturia, Microsystem Design, p.344
Electrophoresis

• Assume an ionic species is carried along in a background electrolyte
• And the carried ion concentration is much less than the background ion concentration
  → Carried ions do not perturb the basic electroosmotic flow profile
  → Carried ions move with the flow at velocity $U_0$

The carried ions will also drift due to the electric field. The drift velocity is equal to

$$v_{ep} = \mu_{ep} E_x$$

$\mu_{ep}$: electrophoretic mobility

Different chemical species have different $\mu_{ep}$
→ Separation of species by electroosmotic flow
→ when $U_0 > v_{ep}$
→ Microchannels by microfabrication
Electrophoretic Separation

- Glass substrate
- Wet etched channels (typically 100 µm wide, 20 µm deep)
- Caped with a flat glass slide
- Channels are filled with an electrolyte buffer solution
- Voltages applied via electrodes immersed in the reservoirs at the ends of each channel.

Ref. Senturia, Microsystem Design, p.345
Electrophoretic Separation

Injection and separation procedure:
1. Positive V1, zero V2
   → Electroosmotic flow carries sample through the junction region
   • Appropriate V3 and V4 must be applied to prevent the sample from entering the horizontal channel, $V_3 > V_J$, $V_4 > V_J$

   $$V_J = \frac{R_2}{R_1 + R_2} V_1$$

   $R_1$ and $R_2$ are electrical resistances.

   $$R_1 = \frac{L_{1,J}}{\sigma_e A_1} \quad R_2 = \frac{L_{2,J}}{\sigma_e A_2}$$

• Clear regions are filled with buffer
• Shaded regions contain sample
**Injection and separation procedure:**

1. Sample injection
   Conditions: Positive $V_1$, zero $V_2$, $V_3 > V_J$, $V_4 > V_J$

2. Sample plug
   Conditions: Large $V_3$, zero $V_4$, $V_1 < V_J$, $V_2 < V_J$
   $\rightarrow$ A sample plug is driven into the separation channel.

3. Separation
   Conditions: same as in step 2.
   $\rightarrow$ Different ionic species separated according to their electrophoretic motilities.
   $\rightarrow$ Fluorescence can be used to observe different species.