Transducers

- Last lecture
  - Piezoelectric
  - Magnetic

- Today:
  - Piezoresistive
  - Pressure sensor

- Reading: Senturia, pp. 469-495.
Resistor as Transducer

⇒ Consider a resistor with dimensions as shown.

\[ R = \frac{\rho L}{A} = \frac{\rho L}{wh} \]

⇒ Effect of small changes in resistor geometry:

\[ dR = \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA + \frac{L}{A} d\rho \]

\[ \frac{dR}{R} = \frac{dL}{L} - \frac{dA}{A} + \frac{d\rho}{\rho} \]

Note, \( \frac{dL}{L} = \varepsilon_{\text{axial}} = \varepsilon \) and

\[ \frac{dR}{R} = (1 + 2\nu)\varepsilon + \frac{d\rho}{\rho} \]

\[ \frac{dA}{A} = \frac{dw}{w} + \frac{dh}{h} = -2\nu\varepsilon \] where \( \nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{axial}}} \).
**Piezoresistive Effect**

- **Gage factor (GF)**
  - Normalized change in resistance with axial strain

  \[ GF = \frac{dR}{R} = \frac{d\rho}{\rho} = (1 + 2\nu) + \frac{\rho}{\epsilon} \]

  \[ GF \text{ for metals } \approx 2 \text{ for } \nu \approx 0.35 \text{ since } \frac{d\rho}{\rho} \text{ is very small.} \]

- **Piezoresistive Effect**

  \[ GF \text{ for semiconductor is very large } \approx 100 \]

  \[ \frac{d\rho}{\rho} \]

  due to large \( \frac{\rho}{\epsilon} \).
Piezoresistive Effect

- **Functional dependence of resistivity**

  By Ohm's Law, $\vec{J} = \sigma \vec{E}$ or $\vec{E} = \rho \vec{J}$ where $\rho = \frac{1}{\sigma}$.

  In 1-D for isotropic solid, this reduces to familiar scalar form: $V = IR$
Piezoresistive Effect

- **Functional dependence of resistivity**

By symmetry, the 9 resistivity coefficients, $\rho_{ij}$, reduce to 6 unique coefficients: $\rho_{11}, \rho_{22}, \rho_{33}, \rho_{23}, \rho_{13},$ and $\rho_{12}$.

If we call these six coefficients:

$\rho_1 = \rho_{11}, \rho_2 = \rho_{22}, \rho_3 = \rho_{33}, \rho_4 = \rho_{23}, \rho_5 = \rho_{13}, \text{ and } \rho_6 = \rho_{12}.$

Then, the resistivity matrix becomes:

\[
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} =
\begin{bmatrix}
\rho_1 & \rho_6 & \rho_5 \\
\rho_6 & \rho_2 & \rho_4 \\
\rho_5 & \rho_4 & \rho_3
\end{bmatrix}
\begin{bmatrix}
J_1 \\
J_2 \\
J_3
\end{bmatrix}
\]
Piezoresistive Effect

- **Piezoresistance in coordinate system aligned to crystal axes**

If the coordinate system is aligned to the \(<100>\) axes of the crystal, \(\rho_{11}, \rho_{22}, \text{ and } \rho_{33}\) relate current density with the electric field in the equivalent \(<100>\) directions.

\[
J_1 = J_{<100>} = \rho_{11} E_{<100>}, \quad J_2 = J_{<010>} = \rho_{22} E_{<010>}, \quad \text{and} \quad J_3 = J_{<001>} = \rho_{33} E_{<001>}. 
\]

- **Unstressed silicon**

\(\rho_{11} = \rho_{22} = \rho_{33} = \rho\) and 
\(\rho_{12} = \rho_{13} = \rho_{23} = 0\)

\[
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} =
\begin{bmatrix}
\rho & 0 & 0 \\
0 & \rho & 0 \\
0 & 0 & \rho
\end{bmatrix}
\begin{bmatrix}
J_1 \\
J_2 \\
J_3
\end{bmatrix}
\]
Piezoresistive Effect

**Piezoresistive material**

In a piezoresistive material, the six resistivity coefficients depend on the stress applied to the material.

\[
\begin{bmatrix}
\rho_1 \\
\rho_2 \\
\rho_3 \\
\rho_4 \\
\rho_5 \\
\rho_6 \\
\end{bmatrix}
= 
\begin{bmatrix}
\rho \\
\rho \\
\rho \\
0 \\
0 \\
0 \\
\end{bmatrix} + 
\begin{bmatrix}
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_3 \\
\Delta \rho_4 \\
\Delta \rho_5 \\
\Delta \rho_6 \\
\end{bmatrix}
\]

- Total resistivity
- Unstressed, quiescent
- \(j\)th resistivity coefficient change due to applied stress in \(j\)th direction
Piezoresistive Effect

**Stressed silicon**

Under the application of a stress for crystalline silicon, the six independent components of stress and strain are related in the following matrix notation.

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{pmatrix}
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{pmatrix}
\]

By symmetry of crystalline silicon, the compliance matrix reduces to the following sparse form:

\[
C =
\begin{pmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{pmatrix}
\]

\[
\sigma_i = \sum_j C_{ij} \epsilon_j \text{ where } i=\{1-6\} \text{ and } j=\{1-6\}
\]
Piezoresistive Effect

- **Stressed silicon**

The six independent resistivity coefficients are related to the six independent stress coefficients through the piezoresistive coefficients:

\[
\begin{bmatrix}
\rho_1 \\
\rho_2 \\
\rho_3 \\
\rho_4 \\
\rho_5 \\
\rho_6
\end{bmatrix}
= \begin{bmatrix}
\rho \\
\rho \\
\rho \\
0 \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_3 \\
\Delta \rho_4 \\
\Delta \rho_5 \\
\Delta \rho_6
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_3 \\
\Delta \rho_4 \\
\Delta \rho_5 \\
\Delta \rho_6
\end{bmatrix}
= \frac{1}{\rho}
\begin{bmatrix}
\pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & \pi_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & \pi_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & \pi_{44}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix}
\]
Piezoresistive Effect

- **Ohm’s law in stressed silicon**

The electric field is related to the current density and the general resistivity as function of stress using

\[
\begin{bmatrix}
E_1 \\
E_2 \\
E_3 \\
\end{bmatrix} = \begin{bmatrix}
\rho_1 & \rho_6 & \rho_5 \\
\rho_6 & \rho_2 & \rho_4 \\
\rho_5 & \rho_4 & \rho_3 \\
\end{bmatrix} \begin{bmatrix}
J_1 \\
J_2 \\
J_3 \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\rho_1 \\
\rho_2 \\
\rho_3 \\
\rho_4 \\
\rho_5 \\
\rho_6 \\
\end{bmatrix} = \begin{bmatrix}
\rho \\
\rho \\
\rho \\
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
\Delta \rho_1 \\
\Delta \rho_2 \\
\Delta \rho_3 \\
\Delta \rho_4 \\
\Delta \rho_5 \\
\Delta \rho_6 \\
\end{bmatrix}
\]

and the piezoresistive coefficients

\[
E_1 = \rho j_1 + \rho \pi_{11} \sigma_{1} j_1 + \rho \pi_{12} (\sigma_2 + \sigma_3) j_1 + \rho \pi_{44} (j_2 \tau_3 + j_3 \tau_2)
\]

\[
E_2 = \rho j_2 + \rho \pi_{11} \sigma_{2} j_2 + \rho \pi_{12} (\sigma_1 + \sigma_3) j_2 + \rho \pi_{44} (j_1 \tau_3 + j_3 \tau_1)
\]

\[
E_3 = \rho j_3 + \rho \pi_{11} \sigma_{3} j_3 + \rho \pi_{12} (\sigma_1 + \sigma_2) j_3 + \rho \pi_{44} (j_1 \tau_2 + j_2 \tau_1)
\]

Unstressed  Normal stress  Transverse stress  Shear stress
Piezoresistive Effect

- **Longitudinal and transverse Piezoresistive coefficients**

  Consider two cases:
  
  (1) Case 1: uniaxial stress in the same direction as current in the transformed coordinate (longitudinal piezoresistive coefficient)
  
  (2) Case 2: uniaxial stress in the perpendicular direction in relation to current in the transformed coordinate (transverse piezoresistive coefficient)

Piezoresistive Coefficients

- **Longitudinal piezoresistive coefficient**

\[
\left( \frac{\Delta \rho}{\rho} \right)_{\text{longitudinal}} = \pi_l \sigma_l
\]

where \( \pi_l = \pi_{11} + 2(\pi_{44} + \pi_{12} - \pi_{11})(l_1^2 m_1^2 + l_1^2 n_1^2 + m_1^2 n_1^2) \)

- **Transverse piezoresistive coefficient**

\[
\left( \frac{\Delta \rho}{\rho} \right)_{\text{transverse}} = \pi_t \sigma_t
\]

where \( \pi_t = \pi_{12} - (\pi_{44} + \pi_{12} - \pi_{11})(l_2^2 m_2^2 + m_2^2 n_2^2 + n_2^2 m_2^2) \)

where \((l_1, m_1, n_1)\) is the set of direction cosines between the longitudinal resistor direction and the crystal axes; \((l_2, m_2, n_2)\) is the set of direction cosines between the transverse resistor direction and the crystal axes

**Example:**
Resistors oriented along [110] directions in (100) wafers:

\[\pi_l = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44}) \text{ and } \pi_t = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})\]
### Piezoresistive Coefficients

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (Ω cm)</th>
<th>$\pi_{11}$ ($10^{-12}$ cm$^2$/dyne or $10^{-11}$ Pa$^{-1}$)</th>
<th>$\pi_{12}$</th>
<th>$\pi_{44}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($p$-type)</td>
<td>7.8</td>
<td>+6.6</td>
<td>-1.1</td>
<td>+138.1</td>
</tr>
<tr>
<td>($n$-type)</td>
<td>11.7</td>
<td>-102.2</td>
<td>+53.4</td>
<td>-13.6</td>
</tr>
<tr>
<td>Germanium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($p$-type)</td>
<td>1.1</td>
<td>-3.7</td>
<td>+3.2</td>
<td>+96.7</td>
</tr>
<tr>
<td>($n$-type)</td>
<td>15.0</td>
<td>-10.6</td>
<td>+5.0</td>
<td>+46.5</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>-2.3</td>
<td>-3.2</td>
<td>-138.1</td>
</tr>
<tr>
<td></td>
<td>5.7</td>
<td>-2.7</td>
<td>-3.9</td>
<td>-136.8</td>
</tr>
<tr>
<td></td>
<td>9.9</td>
<td>-4.7</td>
<td>-5.0</td>
<td>-137.9</td>
</tr>
<tr>
<td></td>
<td>16.6</td>
<td>-5.2</td>
<td>-5.5</td>
<td>-138.7</td>
</tr>
</tbody>
</table>

Ref. Smith, Piezoresistance effect in Germanium and silicon, p. 43-44, 1951.
Gage Factor

- **Gage factor (GF) in semiconductor is a function of**
  - crystal orientation
  - doping type and level
  - temperature
    - temperature coefficient of resistance (T.C.R.)
    - temperature coefficient of gage factor (T.C.G)
  - stress
Crystal Direction Dependence

- The <110> direction has the maximum piezoresistive coefficients for p-type silicon.
- N-type silicon has maxima in the <100>. Magnitude is larger than for p-type silicon.

Ref. Madou, p.202
Doping Dependence (N-type)

- Doping dependence for n-type silicon

\[ \pi(N, T) = P(N, T)\pi(N_{low}, 300K) \]

- Smaller sensitivity at higher temperature.
- Less sensitive to temperature change at higher temperature.

Doping Dependence (P-type)

- Doping dependence for p-type silicon

\[ \pi(N, T) = P(N, T)\pi(N_{low}, 300K) \]

- Smaller sensitivity at higher temperature.
- Less sensitive to temperature change at higher temperature.

Temperature Dependence

Temperature dependence of gage factor:

\[
\frac{\Delta R}{GF} = \frac{R}{\varepsilon} \approx G_0 (1 + \beta \Delta T) \quad \text{neglecting higher order terms where } \beta = \text{temperature coefficient of gage factor (T.C.G.)}
\]

R also depends on temperature as

\[R \approx R_0 (1 + \alpha \Delta T)\]

where \(\alpha = \text{temperature coefficient of resistance (T.C.R)}\)

For the piezoresistance, the temperature dependence of \(\Delta R\) is approximated as:

\[
\Delta R \approx R_0 [1 + \alpha \Delta T + G \varepsilon (1 + (\alpha + \beta)) \Delta T] \varepsilon
\]

Temperature dependence is minimized where \(\alpha + \beta = 0\) around \(10^{18}\) and \(10^{20} \text{ cm}^{-3}\).

**Piezoresistive Pressure Sensor**

- **Resistance change with applied stress**

\[
\frac{\Delta R}{R} \approx \pi_l \sigma_l + \pi_t \sigma_t
\]

- For resistors oriented in the \(<110>\) direction,

\[
\pi_l = \frac{1}{2} (\pi_{11} + \pi_{12} + \pi_{44}) \approx \frac{\pi_{44}}{2}
\]

\[
\pi_t = \frac{1}{2} (\pi_{11} + \pi_{12} - \pi_{44}) \approx -\frac{\pi_{44}}{2}
\]

\[
\frac{\Delta R}{R} \approx \frac{\pi_{44}}{2} (\sigma_l - \sigma_t)
\]

For \(<110>\) resistors in (100) Si wafers,

N-type: \(\pi_l = -31.2\) \(\pi_t = -17.6\)

P-type: \(\pi_l = 71.8\) \(\pi_t = -66.3\) (Units: \(10^{-11}/\text{Pa}\))

\[
R_1 = R_3 = (1 + \pi_l + \nu \pi_t) \sigma_l R_0 = (1 + \alpha_1) R_0
\]

\[
R_2 = R_4 = (1 + \pi_l + \nu \pi_t) \sigma_t R_0 = (1 + \alpha_2) R_0
\]

\[
V_o = \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \approx \frac{\alpha_1 - \alpha_2}{2}
\]

\[
V_s = \frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \approx \frac{\alpha_1 - \alpha_2}{2}
\]
Piezoresistive Pressure Sensor

SOI high-temperature pressure sensor (NovaSensor)

Figure 4.6 Fabrication steps for a piezoresistive gauge, or differential, bulk micromachined pressure sensor.

Ref. Maluf, Introduction to MEMS, p. 106