Lumped-Element Modeling

- Agenda:
  - Design Issues
  - Lumped-Element Modeling
    - Assumptions
    - Conjugate power variables
    - Equivalent circuit modeling
    - One-port elements

- Reading: Senturia, pp. 103-110.
Classification

- **Microsystem “type”:** (determines design strategy)
  - **Technology Demonstration:** (very small volume)
    - drive development activity: (e.g., µTAS, µTurbine, etc.)
      - device proof of concept
      - push fabrication technology
  - **Research Tools:** (small volume, quantitative accuracy)
    - components or complete systems (e.g., AFM tips, Druck pressure sensor)
    - enable research, perform highly specialized task
  - **Commercial Products:** (small to large volume)
    - highly dependent on application/market
    - pressure sensors: MAP versus catheter tip.

**Technology Driven versus Market Driven?**
High-Level Issues

- High-Level Design Issues:
  - **Market:**
    - need, size, time-scale, etc.?
  - **Impact:**
    - enabling, paradigm shift, etc.?
  - **Competition:**
    - competing technologies/organizations?
  - **Technology:**
    - manufacturing requirements, maturity level?
  - **Manufacturing Economics:**
    - cost versus volume.

Relative importance is a function of the type of microsystem!
Detailed Issues

- Detailed Modeling and Design Issues:
  - **System Architecture:**
    - micromachined components, electronics, package, etc.
  - **System Partitioning:**
    - monolithic/hybrid, effects on packaging
  - **Transduction Methods:**
    - effects on cost, performance, and partitioning
  - **Fabrication Technologies:**
    - effects on partitioning
    - robustness/yield issues: repeatability (geometry/materials)
  - **Domain-Specific Knowledge** (all MEMS are multi-energy domain!)
  - **Electronics:**
    - interface, signal conditioning, control, etc.
MEMS Modeling Levels

- **System Level**: block diagrams, lumped elements (ODEs), etc.
- **Device Level**: Energy-based macro-models (ROM).
- **Physical Level**: PDEs, FEM, CFD, etc.
- **Process Level**: Fabrication process modeling (TCAD), masks, etc.

Example

- Capacitive Accelerometer: “multi-energy domain problem”

- **Kinetic Energy**: proof mass motion/inertial domain
- **Potential Energy**: bending strain energy/elastic domain
- **Potential Energy**: electrostatic forces/electrical domain
- **Dissipation**: viscous dissipation/fluid domain
- **Potential Energy**: gas compression/fluid domain
Capacitive Accelerometer continued:

- Complex dynamics problem: (haven’t even considered circuitry yet!)
  - coupled
  - non-linear
  - distributed
  Need efficient, compact, insightful models

Physical Phenomena

Distributed, Non-linear, Coupled PDEs

Lumped Elements ODEs

- Laws
- Continuum?

- Energy-based models for ROM
Macromodels

- Ideal Macromodel:
  - Analytic
  - Accurately captures energy behavior
  - Quasi-static/dynamic
  - Proper dependence on geometry & material properties
  - Agreement with 3-D numerical modeling/experiments on test structures

"oversimplification, but provides physical insight"
Question:

Given the fundamental equations governing the physical phenomena, how do we generate lumped-element models?

- Define ideal elements (single-port elements)
- Define connection laws/KCL/KVL for equivalent circuits
- Develop multiple energy-domain transducers (two-port elements)
Conjugate Power Variables

- **Generation of Lumped Elements:**
  - Now that we know the basic assumption, we will consider the energy exchange between “elements”

\[
P_{BA} \quad \text{and} \quad P_{AB}
\]

- \( P_{AB} \) : Power Flow \( A \rightarrow B \)
- \( P_{BA} \) : Power Flow \( B \rightarrow A \)
Conjugate Power Variables

- **Power Flow:**

\[ P = \frac{dE}{dT} \]; where \( E \) is energy. Both \( P_{AB} \) and \( P_{BA} > 0 \), so let

\[ P_{AB} = u^2 \quad \text{and} \quad P_{BA} = v^2, \]

where both \( u \) and \( v \) are real.

The net power flow from \( A \to B \) is

\[ P_{net} = P_{AB} - P_{BA} = u^2 - v^2 = (u + v)(u - v), \]

so define

\[
\begin{align*}
e & \equiv u + v, \quad \text{EFFORT} \\
 f & \equiv u - v, \quad \text{FLOW}
\end{align*}
\]
**Conjugate Power Variables**

- **Displacement and Momentum:**

  Associated with the EFFORT variable, there is a generalized momentum, \( p \), where

  \[
  p \equiv \int_{t_0}^{t} e(t)dt + p(t_0).
  \]

  Associated with the FLOW variable, there is a generalized displacement, \( q \), where

  \[
  q \equiv \int_{t_0}^{t} f(t)dt + q(t_0).
  \]
Conjugate Power Variables

**Summary:**

Power in an element is defined as $P = e \cdot f$.

The energy in an element is defined as $E = q \cdot e$ or $E = p \cdot f$.

For an IMPEDANCE ANALOGY, $e = Z \cdot f$, where $Z$ is the generalized impedance of the element.

For an ADMITTANCE ANALOGY, $f = Y \cdot e$, where $Y$ is the generalized admittance of the element.
### Examples of Conjugate Power Variables

<table>
<thead>
<tr>
<th>Energy Domain</th>
<th>Effort</th>
<th>Flow</th>
<th>Momentum</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical translation</td>
<td>Force $F$</td>
<td>Velocity $\dot{x}, v$</td>
<td>Momentum $p$</td>
<td>Position $x$</td>
</tr>
<tr>
<td>Fixed-axis rotation</td>
<td>Torque $\tau$</td>
<td>Angular velocity $\omega$</td>
<td>Angular momentum $J$</td>
<td>Angle $\theta$</td>
</tr>
<tr>
<td>Electric circuits</td>
<td>Voltage $V, v$</td>
<td>Current $I, i$</td>
<td>...</td>
<td>Charge $Q$</td>
</tr>
<tr>
<td>Magnetic circuits</td>
<td>MMF $\mathcal{M}$</td>
<td>Flux rate $\dot{\phi}$</td>
<td>...</td>
<td>Flux $\phi$</td>
</tr>
<tr>
<td>Incompressible fluid flow</td>
<td>Pressure $P$</td>
<td>Volumetric flow $Q$</td>
<td>Pressure momentum $\Gamma$</td>
<td>Volume $V$</td>
</tr>
<tr>
<td>Thermal</td>
<td>Temperature $T$</td>
<td>Entropy flow rate $\dot{S}$</td>
<td>...</td>
<td>Entropy $S$</td>
</tr>
</tbody>
</table>

Lumped-Element Modeling

- **Representation:**
  - **Bond Graphs:**
    - high-level, specialized language
    - difficult to learn, but very flexible (i.e., easily handles non-linearities)
  
  - **Equivalent Circuit Elements:**
    - represent elements and interconnects as equivalent circuits
    - Kirchoff’s laws govern behavior
    - easy to learn, but limited to linear passive elements
Single-Port Elements

- **Ideal One-Port Elements:**
  - There are 5 one-port elements
    - directly analogous to electrical elements
    - 2 sources (active)
    - 1 dissipation (passive)
    - 2 energy storage (passive)

- **Sign convention:**
  - We will use the impedance analogy \( e = Z \cdot f \)
    - **effort** is the across variable
    - **flow** is the through variable
    - **power** is entering the element
Source Elements

- **Ideal Flow Source:**
  - Provides a flow equal to $f_0(t)$ for any value of $e$

- **Ideal Effort Source:**
  - Provides an effort equal to $e_0(t)$ for any value of $f$
Generalized Dissipator

**Ideal Dissipator**: “$R$”

Defined directly in terms of $e$ and $f$, $e = e(f)$ or $f = f(e)$ in order to be considered passive, $e = e(f)$ must go through the origin and occupy the first and third quadrants, so that $P = e \cdot f > 0$. 

![Diagram](image)
Generalized Dissipator

- **Ideal Dissipator:** “$R$”
  - Physically represents energy lost by “friction”.
  - No energy storage!

Examples, "$e = Rf$"

- **EQS:** $V = RI$, "Ohm's Law", linear resistor
- **Fluids:** $\Delta p = RQ$, pressure-drop for laminar duct flow (linear)
- **Mechanics:** $F = bu$, linear dash-pot
- **Mechanics:** Columb friction